EDUCATION FOR DEMOCRATIC CITIZENSHIP: THEORY AND TEACHING PRACTICE

Session 1b: CDC in Mathematics Education

By Andy Brittain

In the following text I will be discussing how we can (and why we should) consider, and incorporate, the European Council's Competences for Democratic Culture into maths' lessons.

I will be considering this through four main themes: global systems; communication; numerate thinking and civic mathematics.

When teaching mathematics we need to be aware of how content is being integrated into students' pre-existing mental frameworks. Mathematical procedures can come across as an isolated collection of processes all separately employed as required -rather than as an interlinked and interdependent whole. The towering body of work that constitutes mathematics has been constructed over a very long time in many and varied social and cultural landscapes. It is helpful to point out that the strategies, techniques and approaches we address in the classroom have been selected to enhance problem solving strategies and promote structured and well-reasoned thought. Care needs to be taken to ensure that we are framing mathematical learning in such a way that the rational approaches it espouses can be applied outside of the classroom, to the sociopolitical world that we all inhabit.

Educational guidelines make clear the objective for learners to comprehend mathematical information in a variety of forms as appropriate to the information and to the context. It is then incumbent upon teachers to select contexts that reflect pupils' lives; contexts that provide meaning and depth into their everyday existence. Highlighting the origin of mathematical advances during content delivery encourages the assimilation of information, outward thinking, critical reflection and a greater degree of civic mindedness.







Algebra, for example, has been well studied linking back to the ancient Egyptians and Babylonians, through the Greeks, and into the Islamic world, from which the word Algebra (or al-jabru) is derived. Exploring the challenges met by ancient scholars highlights the need for mathematics in addressing today's difficulties. All those years ago it was as important to know the dimensions required for land development and the interest on a loan as it is today. Although we should stress that intellectual development is attributable to whole societies rather than individuals, teaching pupils about specific contributions, like those of the Persian scholar al-Khwarizmi, is a useful way to personalize their learning. It also enables referencing of characteristics that give a deeper appreciation of the international nature of mathematical knowledge, such as the Indian and Arabic numerals that were blended together to construct our current number system. Promoting greater historico-cultural awareness in the classroom encourages our students to reflect upon the ways they could contribute to society.

Mathematics is rich in history. It is instructive for learners to explore how number systems developed in China, Japan, Central America and other countries. Cultural variation in mathematical methods is still found useful to this day. Some societies use more physically based representations of numbers than others. The abacus is still in common use in places like Russia, China and the Middle East –whereas other countries prefer more abstract approaches such as pen and paper. This immediately raises the question of how different people think. Language is one thing –and students may consider this an unimportant difference– but if someone handles maths differently, then we anticipate a good deal more effort being required. Can it be taken as read that one orange plus one orange is the same as one sheep plus one sheep? A number of cultures use quite different words for numbers in different contexts. After all, the outcome of adding a male to a female sheep might be a little extra sheep. Including this aspect within the educational experience helps learners to consider the views of others.

Other cultural variations that could be explored include notational variants, such as commas instead of decimal points, different number groupings other than multiples of 1000, different problem solving strategies, different gesture correlates and a variety of number base systems. We might also consider the diversity of philosophical views. Our stance may be systematic, but others might prioritize the creative, symbolic or relational aspects. It could come as a surprise to learners that our approach to mathematics has been selected







over centuries, rather than received as a rule of nature. Considering how things could have been different leads to considering how things could become different.

The underlying logical strength of mathematics supports the view that it represents the most effective means of communication. This may be so, in which case it is important to show students how to interpret accurately what is being said – and to effectively deliver their own messages.

Including mathematical content within verbal information, however, often generates semantic difficulties. By the word "even" –do we mean a number divisible by two or a smooth surface? By the word "Series" –do we mean the sum of a sequence of values or a collection of TV programs? It can be a challenge to think of a mathematical word that doesn't also have a non-mathematical meaning.

Drawing attention to this fact by presenting examples of mathematically replete messaging from politicians and other influential voices can provide useful practice. Learners can be encouraged to explore whether the accepted meaning stands up to scrutiny.

This could involve working backwards through the text, from conclusion to assumptions –which may seem unusual at first glance. Students generally work through complexity towards a simpler solution when processing mathematical problems. They aim to arrive at a supportable conclusion from the starting values provided. This approach is good for giving structure to an argument and, when used rhetorically, can convince those listening of the wisdom of a position! But knowing how to do the reverse (reducing a complex set of conclusions to their underlying assumptions) is a very useful skill in participatory democracy.

This process may be a little intimidating, but educators can help their students confront complexity without fear. Society is complex –and it is common to use mathematics to convince populations they should act in a particular way; building the resilience to cope with this is important.

Various forms of media, politicians and many others seek to convince us of the strength of their position. Providing numerical data attributes all the power, certainty and credibility of







mathematics to the communicator –it also makes the case difficult to challenge. Often, careful processing is required but there's not enough time. It may turn out that –following scrutiny– the information is not supported at all! But as exchanges of opinion and points of debate get faster, the processing time gets smaller. This is problematic.

Every day we see numbers floating around without any frame of reference. Small numbers for governments are huge for members of the public. Maths lessons that assist in establishing some sense of scale and proportion would greatly assist with civic engagement. Percentages are frequently cited, but what do they represent in absolute terms? 1% of the population might be referred to in quite an off-hand manner by a politician –when this actually represents hundreds of thousands of people! Learners not only need to do calculations, they need to switch between different representations and reflect upon what each one of them means. We need to encourage our students to have this flexibility in thinking. They should be able to consider whether, say, the average is a good measure of typical income or... maybe the median or modal value is more useful?

Interpreting graphical as well as numerical information is equally important. Graphical formats are frequently used to convey information to large populations. As citizens, it's essential that we effectively assimilate information to function within a modern society, a society in which mathematics governs administrative processes. Learners need to be taught to internalize the processing of information to make well-reasoned and supportable decisions about their community. What opinions do they hold? What action do they want to take –at local and national level? Mathematical thinking helps develop agency.

A few sources are available for graphical data, including government and academic websites, news reports and commercial publications. Assessing what is important to learners, and thereby supplying information they find engaging, is key to stimulating interest in the analysis. Teachers should encourage identification of basic functions within data. This can be through individual plotting of tabulated values or through the scrutiny of pre-plotted graphs.

In terms of preparation for democratic engagement, this exercise would draw attention to the possible ways in which data presentation could be misleading. These would include non-linear or unusual scales, biased data selection or an overly narrow range. Learners







should also be directed towards the origin of the data being examined. How reliable is it? Is it accurate? Do other sources confirm its legitimacy? Is it based on forecasts? Does it conflict with expectations?

In short, students should be taught to undertake critical evaluation.

Developing procedurally sound skills is of course the main objective of any course in mathematics but, as is being discussed, there is also an opportunity to encourage social and democratic engagement.

In the Information Age, procedural approaches need be complimented by careful consideration of the meaning and purpose of data. Mobile phones around the world deliver constant updates: messages that stimulate, motivate, and direct.

Students should be shown how to question why a particular means of representation has been used by those that seek to persuade. For example: why has a very large looking number been displayed instead of one in standard form? Is it being used because it's persuasive in and of itself, rather than because it supports a particular argument? Citizens should have enough knowledge of bar charts, pie charts, histograms, pictograms and scatter graphs to not only understand them, but also to question why an alternative presentation wasn't chosen; Or when faced with a non-mathematical example of propaganda or rhetoric, to wonder why a crisp, objective, mathematical format was not employed.

In order to make use of the data all around them students need to assess correlations and critically consider if causation is both possible and plausible. Trends, if they exist, should be observable, even within somewhat messy data, and cautious extrapolation should be employed to assist with well-reasoned predictions. This requires guided deliberation in lessons in order to become an established skill.

Spotting symmetries and connections is another valuable mathematical ability and helps guide the interpretation of information. The more unintuitive the pattern, the more training is required. We need to use real world illustrations to ensure these skills can be employed as widely as possible. These would include, for example, exponential variations. An appreciation of exponential change is not built into most minds. In modern society, a large







network of interactions can result in this function becoming prominent: spread of disease and the behaviour of financial markets being two clear cases. Individuals need to understand the potential of exponential change in order to consider the actions required. In this way, they can contribute to group decisions that benefit us all. Probability theory is similarly unintuitive. A solid understanding of risk is necessary during every step forward. Citizens should understand the hazards of an enterprise enough to not become overwhelmed, but to move forward with measured, realistic precautions. This enables optimal outcomes to be achieved.

Students learn many valuable mathematical methods that mirror good practice in the democratic process. They should make sure that their initial axioms and starting parameters are well grounded, follow rational principles, explain their working and they should also be aware of the margin for error. This applies to every meaningful pursuit and especially in civic existence.

Collecting similar entities together to be considered as one group for processing purposes is a mathematical technique that benefits communal enterprise. Identifying multiple issues to be addressed as a single category is a powerful strategy. Without the ability to group by type, the panoply of factors influenced and caused by societal living can be overwhelming. Category identification can help select the most effective route forward. Climate change, for example, has had impact along multiple fronts that might look like they require individualized responses. By including them within a common class we explore the potential for a common solution.

Students should be encouraged not only to select appropriate methods and procedures but to reflect upon and critique their own approach: maybe there's a more robust strategy, a quicker route or an alternative interpretation. Maybe someone else is using a more justifiable procedure. Collegiate mathematical enterprise can yield highly positive outcomes. There's no reason mathematicians need to be isolated workers. Construction and refinement of real world models and useful algorithms can very effectively be completed by groups of learners working together.







It cannot be taken as read that systemised thinking will be used by the wider population. To both appreciate the ways interconnected systems and networks link together and to navigate successfully through them requires training.

As the nodes and branches become more webbed and complex, modern bureaucracies become increasingly Kafkaesque.

An awareness of complexity and the confidence to adjust to a more convenient visualization is valuable.

Looking back through time to identify key decisions on local and national policy and to look forward to the end point of the trajectory is important to the establishment of a holistic world view.

Societal thinking and civic duty moves beyond the individual to the many. Collectives need to generalize their concerns in order to act in everybody's interest.

And once democratic action has been taken, the direction of travel should be continually reviewed and logically justified using supportable and rigorous proof. It needs to stand up to continual scrutiny. Representatives must be able to demonstrate that they have a well-reasoned and rational approach –and their supporters should know to demand that of them.

Learners must be taught to have high expectations. Although the level of complexity in a society necessitates some conjecture, we all have a right to expect messaging to be clear, cohesive and thorough. We all need to be able to identify, as best as possible, which of the pathways presented is the most positive. Once a course of action is decided upon by all, and as long as it continues to stand up to scrutiny, the group should expect commitment and perseverance along the chosen route, as with solving any problem in mathematics.

In the foregoing, I have explored how mathematics lessons could (and should) incorporate the European Council's Competences for Democratic Culture. We considered the benefits of encouraging students to view mathematics in a global context. We examined the importance of critically evaluating mathematical messaging. We raised the sociological







value of mathematics and we looked at how mathematical thinking is necessary to secure and legitimize policy decisions.

In order for modern democracies to thrive, their populations need to be empowered and encouraged to think critically about society and their role within it. As educators, we seek to produce confident, competent and critical mathematical thinkers. To achieve this, we should look at the full potential of the curriculum. Look beyond the procedures to recognize that mathematics is a way to look at the world that supports full and active democratic participation.





